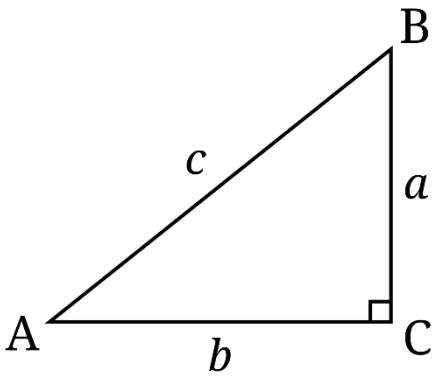
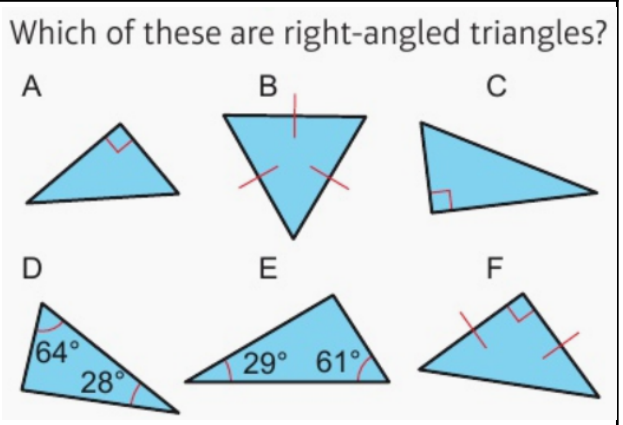
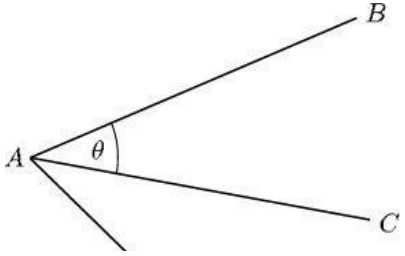
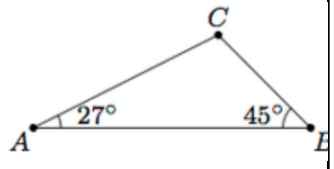
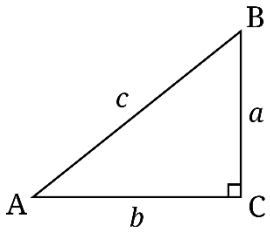
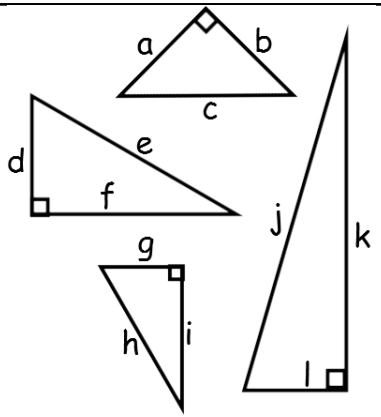
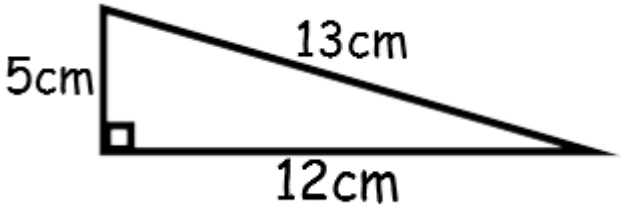
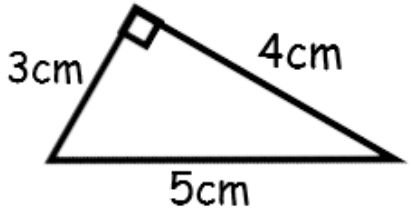
**GCSE Mathematics – Unit 12: Right-Angled Triangles** **Name:……**

| Topic/Skill | Definition/Tips | Example | Your Turn |
|---|---|---|---|
| **Right-Angled Triangle** | Any triangle with an angle of 90°. With triangles, we normally label **corners with capital** letters, and the **opposite length with the same letter lower-case.**<br><br>**Don't assume that triangles have right-angles just because it looks like they do.** Examiners will either tell you with a little square in the corner, or **you will be able to work it out.** |  |  |
| **Angle Notation** | You can use **one lower-case** letter, eg. θ (called theta) or $x$<br><br>We can use the symbol ∠ as a shorthand for angle<br>You can use **three upper-case** letters, eg. $BAC$ – where the middle letter is the corner of the angle. |  <br>∠BAC is θ ° | How many degrees is:<br>∠BAC: ……<br>∠ABC: ……<br>∠ACB: ……<br>  |

<table>
<tr>
<td>Hypotenuse</td>
<td>In a Right-Angled Triangle the hypotenuse is always the longest side. It is always opposite the 90° angle.</td>
<td> 

Side c is the Hypotenuse, opposite ∠ACB which is 90°</td>
<td> 

Which sides are hypotenuses?</td>
</tr>
<tr>
<td>Pythagoras' Theorem</td>
<td>2500 years ago, a Greek Mathematician discovered that in any right-angled triangle, the square of the hypotenuse is the same as the square of the other two sides added together.<br>$c^2 = a^2 + b^2$<br>These letters assume that c is the hypotenuse.</td>
<td> 

$13^2 = 5^2 + 12^2$<br>$169 = 25 + 144$</td>
<td> 
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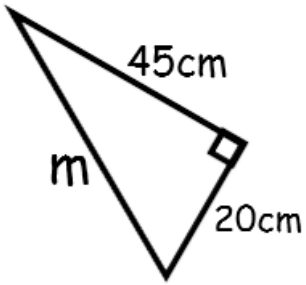
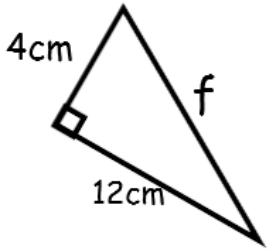
<tr>
<td>Pythagorean Triple</td>
<td>It is rare that 3 lengths in a right-angled triangle will all be whole numbers (integers).<br>Any 3 integers that can go in a right-angled triangle are called a Pythagorean Triple.</td>
<td>$3^2 + 4^2 = 5^2$<br>$5^2 + 12^2 = 13^2$<br>$7^2 + 24^2 + 25^2$<br>…</td>
<td>See if you can find your own triple.<br>Hint: what happens if you multiply all 3 sides by the same number? Test your answer using the Theorem.<br><br>Can you explain why this worked?</td>
</tr>
<tr>
<td colspan="4">Finding missing lengths: always start by writing out the Theorem and then substitute in the lengths you have</td>
</tr>
</table>

<table>
<tr>
<td>Find the Hypotenuse</td>
<td>Add the squares of the shorter two sides and then find the square root.</td>
<td>$c^2 = a^2 + b^2$<br> 

$m^2 = 20^2 + 45^2$<br>$m^2 = 400 + 2025$<br>$m^2 = 2425$<br>$m = \sqrt{2425}$<br>$m = 49.24428...$<br>$m \approx 49.2\text{cm}$</td>
<td> 
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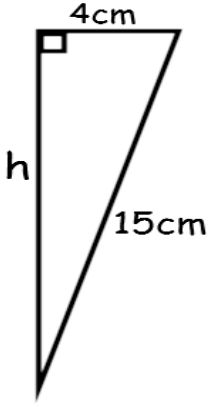
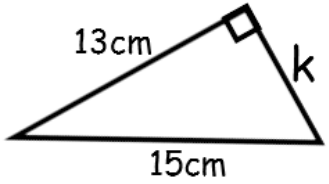
<tr>
<td>Find the shorter side</td>
<td>Subtract the square of a shorter side from the square of the hypotenuse and then find the square root.</td>
<td> 

$c^2 = a^2 + b^2$<br>$15^2 = h^2 + 4^2$<br>$225 = h^2 + 16$<br>$209 = h^2$<br>$h = \sqrt{209}$<br>$h = 14.456832...$<br>$h \approx 14.5cm$</td>
<td> 
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<tr>
<td colspan="3">Trigonometry: ‘trigo-’ means triangle, ‘-metry’ means measure.<br>We use it to find missing lengths and angles in Right-Angled Triangles.</td>
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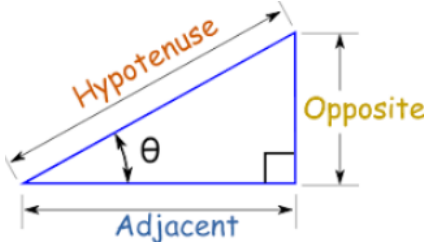
<tr>
<td>Opposite, Adjacent and Hypotenuse</td>
<td> 
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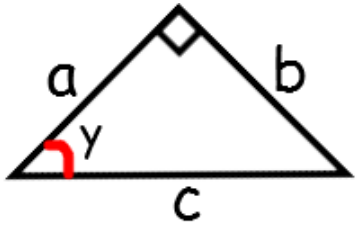
<td> 
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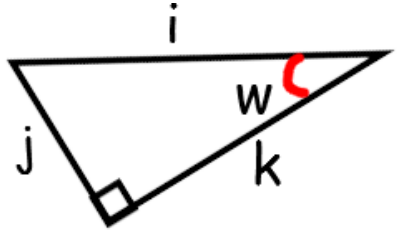
<td> 
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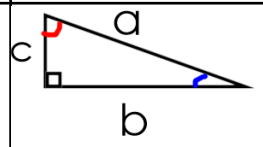
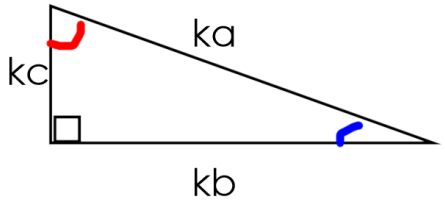
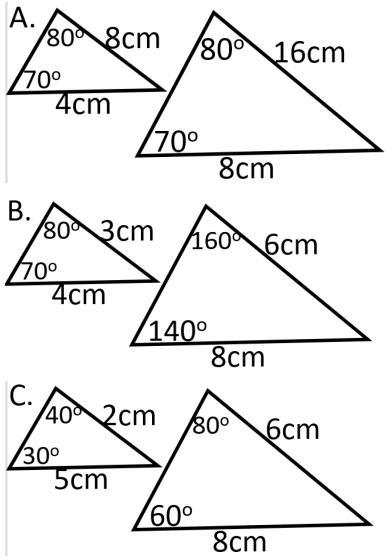
|---|---|---|---|
| When we talk about sides in trigonometry it always depends on the angle we are focussing on. | We know the Hypotenuse.<br>The Opposite is the side opposite the Angle<br>The Adjacent is the side **next to** the angle (not the hypotenuse) | If we are working with angle y, then:<br>C is the hypotenuse<br>**A is the adjacent because it is next to Y**<br>B is the opposite | If we are with angle w<br><br>**Side k** is the…<br><br>**i** is the…<br><br>**j** is the… |
| **Similarity** | Similarity is when two shapes have the same **proportions.**<br>The sides share the **same ratio**, but might all be twice as long or 3 times as long or k times as long.<br>**All the angles are the same**.<br>In other words one shape is an **enlargement** of the other! |  <br>All the sides in the second triangle are **k** times longer than the sides in the first triangle, but **the angles are the same.**<br>  | Similar, not similar, or not enough info?<br>  |

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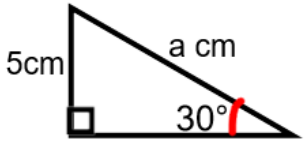
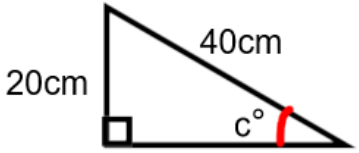
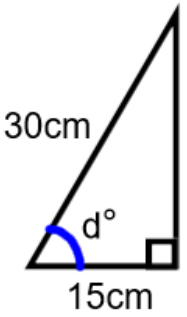
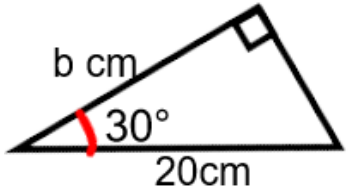
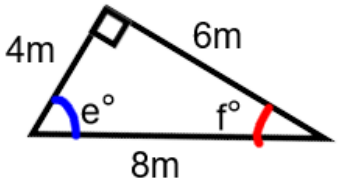
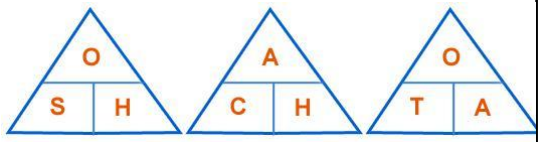
<tr>
<td>Ratios in Triangles</td>
<td>Similar triangles have the same ratios between their sides.<br>We can use this to find missing lengths.<br>We call these Trigonometric ratios Sine, Cosine and Tangent<br><br>The Sine ratio of a R.A. triangle with 30° is $\frac{Opp}{Hyp} = \frac{1}{2}$ : the Hypotenuse is double the Opposite.<br>If two triangles have the same ratios of lengths, you know they have the same angles.</td>
<td>In a Right-Angled Triangle with one angle of 30°, the opposite angle is always double the adjacent.<br>$Sin30° = \frac{1}{2}$<br> 

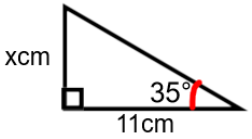
a = 10cm<br><br> 

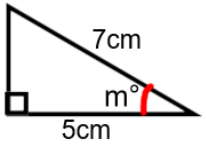
The hypotenuse is double the opposite, so c=30°</td>
<td> 

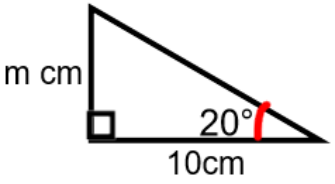


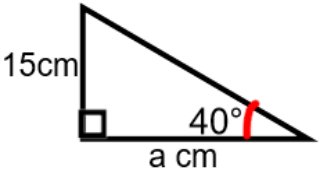
b=……<br> 

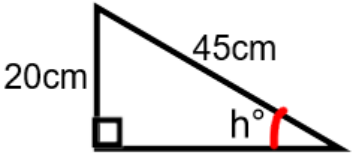
e=……<br>f=……<br><br>d=…..</td>
</tr>
<tr>
<td>Trigonometric Formulae to find a missing length<br><br>When finding a missing angle, use the ‘inverse’ trigonometric function by pressing</td>
<td>Use SOHCAHTOA.<br><br>$\sin sin\ \theta = \frac{O}{H}$<br><br>$\cos cos\ \theta = \frac{A}{H}$<br><br>$\tan tan\ \theta = \frac{O}{A}$<br><br> 

Label the sides of the triangle.<br>Use SOH CAH TOA to decide which ratio to use.</td>
<td> 

Use ‘Opposite’ and ‘Adjacent’, so use ‘tan’<br>$\tan tan\ 35 = \frac{x}{11}$<br>$x = 11 \tan tan\ 35 = 7.70cm$<br>---<br> 

Use ‘Adjacent’ and ‘Hypotenuse’, so use ‘cos’<br>$\cos cos\ m = \frac{5}{7}$</td>
<td> 

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| the 'shift' button on the calculator. | Then **substitute** and solve. | $m = cos^{-1}\left(\frac{5}{7}\right) = 44.4°$ | |