The Bridge to A level

Test Yourself Worked Solutions

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1   <u>Solving quadratic equations</u>

Question 1

Find the real roots of the equation $x^4 - 5x^2 - 36 = 0$ by considering it as a quadratic equation in x^2.

Treat as a quadratic in x^2.

Factorise $(x^2 - 9)(x^2 + 4) = 0$

$\rightarrow$ $(x^2 - 9) = 0$ or $(x^2 + 4) = 0$

$\rightarrow$ $x^2 = 9$ or $x^2 = -4$

$\rightarrow$ $x = \pm 3$ or   No real roots

$\rightarrow$ $x = \pm 3$

(4)

Question 2

(i)   Write $4x^2 - 24x + 27$ in the form of $a(x - b)^2 + c$

$$4x^2 - 24x + 27$$
$$= 4(x^2 - 6x) + 27 \qquad \text{Don't take the factor of 4 out of the constant.}$$
$$= 4\left[(x - 3)^2 - 9\right] + 27$$
$$= 4(x - 3)^2 - 36 + 27$$
$$= 4(x - 3)^2 - 9$$

(4)

(ii)   State the coordinates of the minimum point on the curve $y = 4x^2 - 24x + 27$.

Minimum point at (3,-9)

(2)

Total / 10

2   **Changing the Subject**

Question 1

Make t the subject of the formula $s = \frac{1}{2}at^2$

$$s = \frac{1}{2}at^2$$

$$2s = at^2$$

$$\frac{2s}{a} = t^2$$

$$t = \pm\sqrt{\frac{2s}{a}}$$

(3)

Question 2

Make x the subject of $3x - 5y = y - mx$

$$3x - 5y = y - mx$$

$$3x + mx - 5y = y$$

$$3x + mx = y + 5y$$

$$x(3+m) = 6y$$

$$x = \frac{6y}{(3+m)}$$

Get to terms with x = terms without x.

Factorise, then divide by factor

(3)

Question 3

Make x the subject of the equation $y = \frac{x+3}{x-2}$

$$y = \frac{x+3}{x-2}$$

$$y(x-2) = x+3$$

$$xy - 2y = x+3$$

$$xy - x = 2y+3$$

$$x(y-1) = 2y+3$$

$$x = \frac{2y+3}{y-1}$$

(4)

Total / 10

3

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3   <u>Simultaneous equations</u>

Question 1

Find the coordinates of the point of intersection of the lines $\quad x + 2y = 5$ and $y = 5x - 1$

$x + 2(5x-1) = 5$
$x + 10x - 2 = 5$
$11x = 7$
$x = \frac{7}{11} \qquad\qquad y = \frac{35}{11} - 1 \qquad y = \frac{24}{11}$

(3)

Question 2

The lines $y = 5x - a$ and $y = 2x + 18$ meet at the point $(7, b)$. Find the values of a and b.

$5x - a = 2x + 18$
$35 - a = 14 + 18$
$a = 3 \quad b = 35 - 3 = 32$

(3)

Question 3

A line and a curve has the following equations :

$$3x + 2y = 7 \qquad\qquad\qquad y = x^2 - 2x + 3$$

Find the coordinates of the points of intersection of the line and the curve by solving these simultaneous equations algebraically

Substitute y from 2nd equation into 1st.

$3x + 2(x^2 - 2x + 3) = 7$

$3x + 2x^2 - 4x + 6 = 7$

$2x^2 - x + 6 = 7$

$2x^2 - x - 1 = 0$

Factorise : $(2x + 1)(x - 1) = 0$

means $\quad 2x + 1 = 0 \quad$ or $\quad x - 1 = 0$

$\qquad\qquad 2x = -1 \qquad$ or $\quad x = 1$

$\qquad\qquad x = -\frac{1}{2}$

when $x = -\frac{1}{2}$:

$(3x - \frac{1}{2}) + 2y = 7$

$-1.5 + 2y = 7$

$2y = 8.5$

$y = 4.25$

when $x = 1$

$(3 \times 1) + 2y = 7$

$3 + 2y = 7$

$2y = 4$

$y = 2$

First point $(-\frac{1}{2}, 4.25)$

Second point $(1, 2)$.

(4)

4 <u>Surds</u>

Question 1

(i) Simplify $\sqrt{24} + \sqrt{6}$

(i) $\sqrt{24} + \sqrt{6}$

$= \sqrt{4}\sqrt{6} + \sqrt{6}$

$= 2\sqrt{6} + \sqrt{6}$

$= 3\sqrt{6}$

(2)

(ii) Express $\dfrac{36}{5-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers.

(ii) $\dfrac{36}{5-\sqrt{7}}$ $= \dfrac{36}{(5-\sqrt{7})} \times \dfrac{(5+\sqrt{7})}{(5+\sqrt{7})}$

TO RATIONALISE THE DENOMINATOR of $(x - \sqrt{y})$ top + multiply bottom by $(x + \sqrt{y})$

$= \dfrac{36 \times (5+\sqrt{7})}{25 - 5\sqrt{7} + 5\sqrt{7} - (\sqrt{7})^2}$

$= \dfrac{36(5+\sqrt{7})}{18}$

$= 2(5+\sqrt{7})$

$= 10 + 2\sqrt{7}$

(3)

Question 2

(i) Simplify $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$

$+i)$ $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$

$= 30\sqrt{6} - \sqrt{4}\sqrt{6}$

$= 28\sqrt{6}$

(2)

(ii) Express $(2 - 3\sqrt{5})^2$ in the form $a + b\sqrt{5}$, where a and b are integers.

$(2-3\sqrt{5})(2-3\sqrt{5}) = 4 - 6\sqrt{5} - 6\sqrt{5} + 9\times5$

$= 49 - 12\sqrt{5}.$ (3)

(3)

5 <u>Indices</u>

Question 1

Find the value of the following.

(i) $\left(\frac{1}{3}\right)^{-2}$

$$\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^{2}$$
$$= 3^2$$
$$= 9$$

(2)

(ii) $16^{\frac{3}{4}}$

$$16^{3/4} = \left(16^{\frac{1}{4}}\right)^3$$
$$= (2)^3$$
$$= 8$$

(2)

Question 2

(i) Find a, given that $a^3 = 64x^{12}y^3$

$$a^3 = 64 x^{12} y^3$$
$$= 4^3 (x^4)^3 y^3$$
$$= (4x^4 y)^3$$
$$\Rightarrow a = 4x^4 y$$

(2)

(ii) $\left(\frac{1}{2}\right)^{-5}$

$$\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5$$
$$= 2^5$$
$$= 32$$

Negative indise
invets the
fraction

(2)

Question 3

Simplify $\dfrac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$

$$\frac{16^{\frac{1}{2}}}{81^{3/4}} = \frac{\sqrt{16}}{\sqrt[4]{81^3}} = \frac{4}{3^3} = \frac{4}{27}$$

(2)

6   <u>Properties of Lines</u>

Question 1

The points A (-1,6), B (1,0) and C (13,4) are joined by straight lines. Prove that AB and BC are perpendicular.

Grad of AB   = $\frac{0-6}{1--1}$ =   -3

Grad of BC   = $\frac{4-0}{13-1}$ = $\frac{1}{3}$

Product of gradients is   =   -3 x $\frac{1}{3}$ = -1.   Hence AB and BC are perpendicular.

(2)

Question 2

A and B are points with coordinates (-1,4) and (7,8) respectively. Find the coordinates of the midpoint, M, of AB.

Midpoint is $(\frac{7+-1}{2}, \frac{8+4}{2})$ = (3, 6)

(1)

Question 3

A line has gradient -4 and passes through the point (2,-6). Find the coordinates of its points of intersection with the axes.

Equation of line is (y - -6) = -4(x – 2)   ie   y = -4x + 2

x = 0 →   y = 2   Coordinates (0,14)
y = 0 →   x = 0.5   Coordinates (3.5,0)

(4)

Question 4

Find the equation of the line which is parallel to y = 3x + 1 and which passes through the point with coordinates (4,5).

Gradient 3

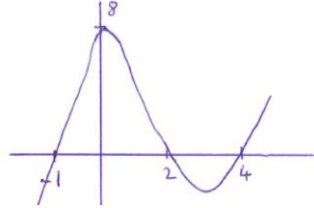   (y – 5) = 3(x – 4)

→   y = 3x - 7

(3)

Total / 10

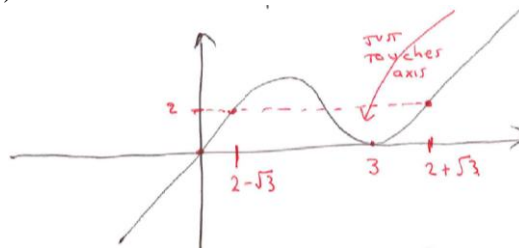# 7 Sketching curves

Question 1

You are given that $f(x) = (x + 1)(x - 2)(x - 4)$. Sketch the graph of $y = f(x)$
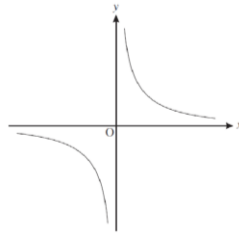

(3)

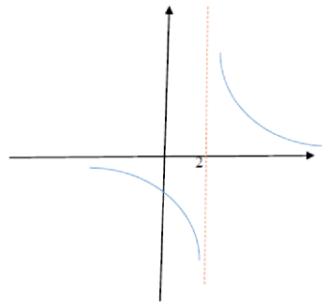**Question 2**

Sketch the graph of $y = x(x - 3)^2$

(3)

Question 3

This diagram shows a sketch of the graph of $y = \frac{1}{x}$

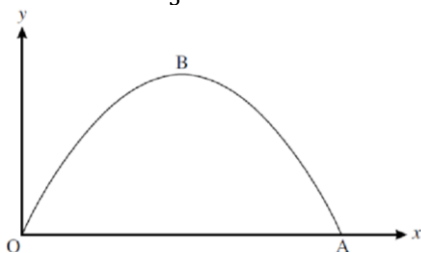Sketch the graph of $y = \frac{1}{x-2}$, showing clearly any points where it crosses the axes.

(0,-0.5)   (3)

Question 4

This curve has equation $y = \frac{1}{5}x(10 - x)$. State the value of x at the point A.

$$y = \frac{x}{5}(10 - x)$$

$$x = 10$$

(1)

8   **Transformation of functions**

Question 1

The graph of $y = x2 - 8x + 25$ is translated by $\begin{pmatrix} 0 & -20 \end{pmatrix}$. State an equation for the resultant graph.

$$y = x^2 - 8x + 25 - 20$$

$$\Rightarrow \quad y = x^2 - 8x + 5$$

(1)

Question 2

$f(x) = x^3 - 5x + 2$

Show that $f(x - 3) = x^3 - 9x^2 + 22x - 10$

$$f(x - 3) = (x - 3)^3 - 5(x - 3) + 2$$
$$= (x^2 - 6x + 9)(x - 3) - 5x + 15 + 2$$
$$= x^3 - 3x^2 - 6x^2 + 18x + 9x - 27 - 5x + 15 + 2$$
$$= x^3 - 9x^2 + 22x - 10$$

(4)

Question 3

You are given that $f(x) = 2x^3 + 7x^2 - 7x - 12$

Show that $f(x - 4) = 2x^3 - 17x^2 + 33x$

$$f(x - 4) = (x - 4 + 4)(2(x - 4) - 3)(x - 4 + 1)$$
$$= x(2x - 8 - 3)(x - 3)$$
$$= x(2x - 11)(x - 3)$$
$$= x(2x^2 - 11x - 6x + 33)$$
$$= 2x^3 - 17x^2 + 33x$$

(3)

Question 4

You are given that $f(x) = (x + 1)(x - 2)(x - 4)$. The graph of $y = f(x)$ is translated by $\begin{pmatrix} 3 & 0 \end{pmatrix}$.

State an equation for the resulting graph. You need not simplify your answer.
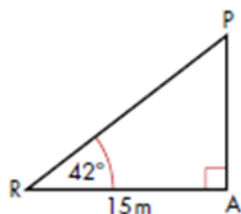
(2)

$(x + 1 - 3)(x - 2 - 3)(x - 4 - 3)$
ie $(x - 2)(x - 5)(x - 7)$

Total / 10

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9 <u>Trigonometric ratios</u>

Question 1

AP is a telephone pole. The angle of elevation of the top of the pole from the point R on the ground is 42° as seen in the diagram.

Calculate the height of the pole. Give your answer to 3 significant figures.

Tan 42° $= \frac{opp}{adj}$ (M1)

$0.9004 = \frac{height\ of\ pole}{15}$ (M1)

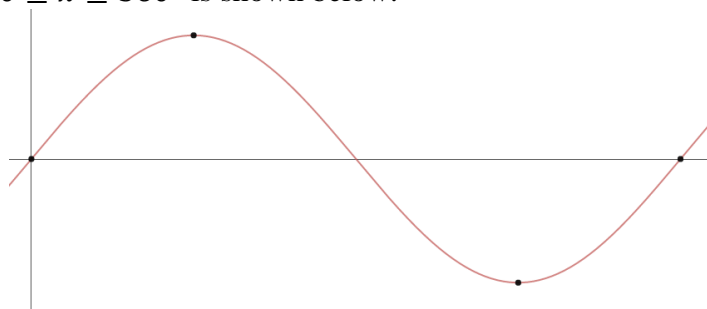13.5(06) m = height of pole   (A1)   (3)

Question 2

Given that sin Θ $= \frac{\sqrt{3}}{4}$, find in surd form the possible values of cos Θ.
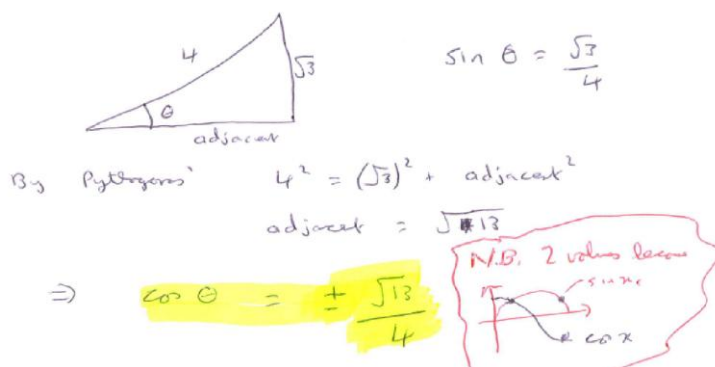
(3)

Question 3

The graph of $y = \sin x$ for $0 \le x \le 360°$ is shown below.

What are the coordinates of the 4 points labelled on the graph?

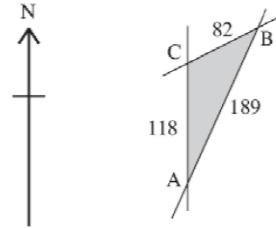(……0…, …0…)
(90…, ……1…)
(270…, …-1……)
(360…, ……0…)
(4)

10   <u>Sine / Cosine Rule</u>

Question 1

This diagram shows a village green which is bordered by 3 straight roads AB, BC and CA. The road AC runs due North and the measurements are shown in metres.

Not to scale

(i)   Calculate the bearing of B from C, giving your answer to the nearest $0.1°$

$$\cos C = \frac{82^2 + 118^2 - 189^2}{2 \times 82 \times 118} = -0.778$$

$$C = \cos^{-1}(-0.778)$$

$$= 141.1°$$

$$\text{Bearing} = 180 - 141.1° = \boxed{38.9°}$$

(4)

(ii)   Calculate the area of the village green.

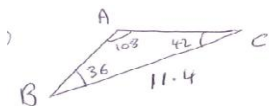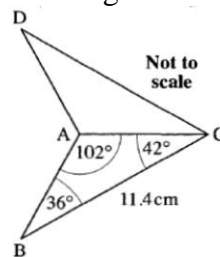$$\text{Area} = \frac{1}{2} \, bc \, \sin C$$

$$= \frac{1}{2} \times 82 \times 118 \times \sin 141.1°$$

$$= \boxed{3032}$$

(2)

Question 2

This diagram shows a logo ABCD. It is symmetrical about AC.
Find the length of AB and hence find the area of the logo

Not to scale

By the Sine Rule,

$$\frac{AB}{\sin 42} = \frac{11.4}{\sin 108}$$

$$\Rightarrow \boxed{AB} = \frac{11.4 \times \sin 42}{\sin 108} = \boxed{7.798}$$

$$\boxed{\text{Area of logo}} = 2 \times \text{Area of Triangle}$$

$$= 2 \times \frac{1}{2} \, ac \, \sin B$$

$$= 11.4 \times 7.798 \times \sin 36$$

$$= \boxed{52.2}$$

(4)

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Total / 10