The Bridge to A level

Diagnosis Worked Solutions

1 Solving quadratic equations

Question 1

Solve $x^2 + 6x + 8 = 0$

$(x + 2)(x + 4) = 0$

$x = -2$ or -4

(2)

Question 2

Solve the equation $y^2 - 7y + 12 = 0$

Hence solve the equation $x^4 - 7x^2 + 12 = 0$

$y^2 - 7y + 12 = 0$

$(y-3)(y-4) = 0 \rightarrow y = 3$ or $y = 4$

$x^4 - 7x^2 + 12 = 0 \rightarrow$ let $x^2 = y$

$(x^2)^2 - 7x^2 + 12 = 0 \rightarrow y^2 - 7y + 12 = 0 \rightarrow y = 3$ or $y = 4$

$\rightarrow x^2 = 3$ or $x^2 = 4$

$\rightarrow x = \pm\sqrt{3}$ or $x = \pm 2$

(4)

Question 3

(i) Express $x^2 - 6x + 2$ in the form $(x-a)^2 - b$

$x^2 - 6x + 2 = (x-3)^2 - 9 + 2$

$= (x-3)^2 - 7$

(3)

(ii) State the coordinates of the minimum value on the graph of $y = x^2 - 6x + 2$

Minimum point of $x^2 - 6x + 2$ is therefore $(3, -7)$

(1)

Total / 10

2 Changing the subject

Question 1

Make v the subject of the formula $E = \frac{1}{2}mv^2$

$$E = \frac{1}{2}mv^2$$

$$\Rightarrow 2E = mv^2$$

$$\Rightarrow \frac{2E}{m} = v^2$$

$$\pm\sqrt{\frac{2E}{m}} = v$$

(3)

Question 2

Make r the subject of the formula $V = \frac{4}{3}\pi r^2$

$$V = \frac{4}{3}\pi r^3$$ get rid of fractions

$$3V = 4\pi r^3$$ make r^3 the subject.

$$\frac{3V}{4\pi} = r^3$$ cube both sides

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

(3)

Question 3

Make c the subject of the formula $P = \frac{c}{c+4}$

$$P = \frac{c}{c+4}$$ Get rid of fractions

$$\Rightarrow P(c+4) = c$$ Expand brackets

$$\Rightarrow Pc + 4P = c$$ Get terms with c on L.H.S., other terms on R.H.S.

$$Pc + 4P - c = 0$$

$$Pc - c = -4P$$ Factorise.

$$c(P-1) = -4P$$ Divide

$$c = \frac{-4P}{P-1} \quad \left(= \frac{4P}{1-P}\right)$$

(4)

Total / 10

PiXL Partners in excellence

3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines y = 3x + 1 and x + 3y = 6

$y = 3x+1$ and $x+3y=6$

$x + 3(3x+1) = 6$

$x + 9x + 3 = 6$

$10x = 3$

$x = \frac{3}{10}$

$y = 3\left(\frac{3}{10}\right)+1$

$= \frac{9}{10}+1$

$= 1\frac{9}{10}$

(3)

$\left(3/10,\ 1\,9/10\right)$ or $(0.3,\ 1.9)$

(3)

Question 2

Find the coordinates of the point of intersection of the lines 5x + 2y = 20 and y = 5 – x

(ii) $5x + 2y = 20$ & $y = 5 - x$.

Solving simultaneously

$5x + 2(5-x) = 20$ M1

$\Rightarrow 5x + 10 - 2x = 20$

$\Rightarrow 3x = 10$

$\Rightarrow x = 10/3$, $y = 5 - 10/3 = 5/3$

Pt. of intersection is $(10/3,\ 5/3)$ A2

Note – if you round these fractions to decimals (10.3, 1.7) you lose a mark.

(3)

Question 3

Solve the simultaneous equations

$x^2 + y^2 = 5$ $y = 3x + 1$

Sub in $y = 3x+1$ into equation 2.

$x^2 + (3x+1)^2 = 5$

$x^2 + (3x+1)(3x+1) = 5$

$x^2 + 9x^2 + 3x + 3x + 1 = 5$

$10x^2 + 6x + 1 = 5$

$10x^2 + 6x - 4 = 0$

$(\div 2)$

$5x^2 + 3x - 2 = 0$

$(5x-2)(x+1) = 0$

$x = \frac{2}{5}$ or $x = -1$

when $x = \frac{2}{5}$

$y = \left(3 \times \frac{2}{5}\right) + 1$

$= \frac{6}{5} + \frac{5}{5} = \frac{11}{5}$

when $x = -1$

$y = (3 \times -1) + 1$

$= -3 + 1$

$= -2$

(4)

Total / 10

4 **Surds**

Question 1

(i) Simplify $(3 + \sqrt{2})(3 - \sqrt{2})$

$(3+\sqrt{2})(3-\sqrt{2})$

$= 3^2 + 3\sqrt{2} - 3\sqrt{2} - (\sqrt{2})^2$

$= 9 - 2$

$= 7$

(2)

(ii) Express $\frac{1+\sqrt{2}}{3-\sqrt{2}}$ in the form $a + b\sqrt{2}$ where a and b are rational

$\frac{(1+\sqrt{2})}{(3-\sqrt{2})} = \frac{(1+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$

$= \frac{3 + \sqrt{2} + 3\sqrt{2} + (\sqrt{2})^2}{7}$

$= \frac{3 + 4\sqrt{2} + 2}{7}$

$= \frac{5}{7} + \frac{4}{7}\sqrt{2}$

To rationalise a denominator of form $(x + \sqrt{y})$ multiply top + bottom by $(x - \sqrt{y})$

(3)

Question 2

(i) Simplify $5\sqrt{8} + 4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$ where a and b are integers and b is as small as possible.

(i) $5\sqrt{8} + 4\sqrt{50}$

$= 5\sqrt{4}\sqrt{2} + 4\sqrt{25}\sqrt{2}$

$= 5 \times 2\sqrt{2} + 4 \times 5\sqrt{2}$

$= 10\sqrt{2} + 20\sqrt{2}$

$= 30\sqrt{2}$

Always look for square number factors

(2)

(ii) Express $\frac{\sqrt{3}}{6-\sqrt{3}}$ in the form $p + q\sqrt{3}$ where p and q are rational

$\frac{\sqrt{3}}{6-\sqrt{3}} = \frac{\sqrt{3}}{6-\sqrt{3}} \times \frac{(6+\sqrt{3})}{(6+\sqrt{3})}$

$= \frac{\sqrt{3} \times 6 + \sqrt{3}\sqrt{3}}{6^2 - (\sqrt{3})^2}$

$= \frac{6\sqrt{3} + 3}{36 - 3}$

$= \frac{3 + 6\sqrt{3}}{33}$

$= \frac{3}{33} + \frac{6}{33}\sqrt{3}$

$= \frac{1}{11} + \frac{2}{11}\sqrt{3}.$

(3)

Total / 10

5 Indices

Question 1

Simplify the following

(i) a^0 (1)

(ii) $a^6 \div a^{-2}$ (1)

(iii) $(9a^6b^2)^{-0.5}$ (3)

(i) $a^0 = 1$

(ii) $a^6 \div a^{-2} = a^{6--2}$

$= a^8$

(iii) $(9a^6b^2)^{-1/2} = (3^2a^6b^2)^{-1/2}$

$= 3^{-1}a^{-3}b^{-1}$ $\left(= \frac{1}{3a^3b} \right)$

Question 2

(i) Find the value of $\left(\frac{1}{25}\right)^{-0.5}$ (2)

(ii) Simplify $\frac{(2x^2y^3z)^5}{4y^2z}$ (3)

i) $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = (25)^{\frac{1}{2}} = \sqrt{25} = \pm 5$

ii) $\frac{(2x^2y^3z)^5}{4y^2z} = \frac{2^5x^{10}y^{15}z^5}{2^2y^2z^1}$

$= 2^{5-2}x^{10}y^{15-2}z^{5-1}$

$= 2^3x^{10}y^{13}z^4 = 8x^{10}y^{13}z^4$

Total / 10

PiXL Partners in excellence

6 Properties of Lines

Question 1

A (0,2), B (7,9) and C (6,10) are three points.

(i) Show that AB and BC are perpendicular

Grad of AB = $\frac{9-2}{7-0}$ = 1

Grad of BC = $\frac{10-9}{6-7}$ = -1

Product of gradients = 1 x -1 = -1 → AB and BC perpendicular

(3)

(ii) Find the length of AC

$(6-0)^2 + (10-2)^2 = AC^2$
AC = 10

(2)

Question 2

Find, in the form y = mx + c, the equation of the line passing through A (3,7) and B (5,-1).
Show that the midpoint of AB lies on the line x + 2y = 10

$m = \frac{-1-7}{5-3} = -\frac{8}{2} = -4$

$y = -4x + c$

Substitute in (3,7) [5,-1] would do equally as well

$7 = -4 \times 3 + c$

$\Rightarrow 19 = c$

$\Rightarrow y = -4x + 19$

Midpoint of AB = (4, 3)

Sub. in to $x + 2y = 10$ & show that equation is true.

$4 + 2 \times 3 = 4 + 6 = 10$

TRUE.

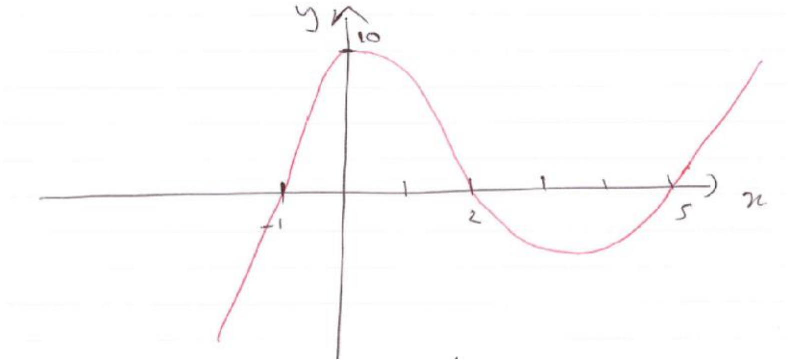
(5)

Total / 10

7 Sketching curves

Question 1

In the cubic polynomial f(x), the coefficient of x^3 is 1. The roots of f(x) = 0 are -1, 2 and 5.
Sketch the graph of y = f(x)



(3)

Question 2

Sketch the graph of $y = 9 - x^2$

9

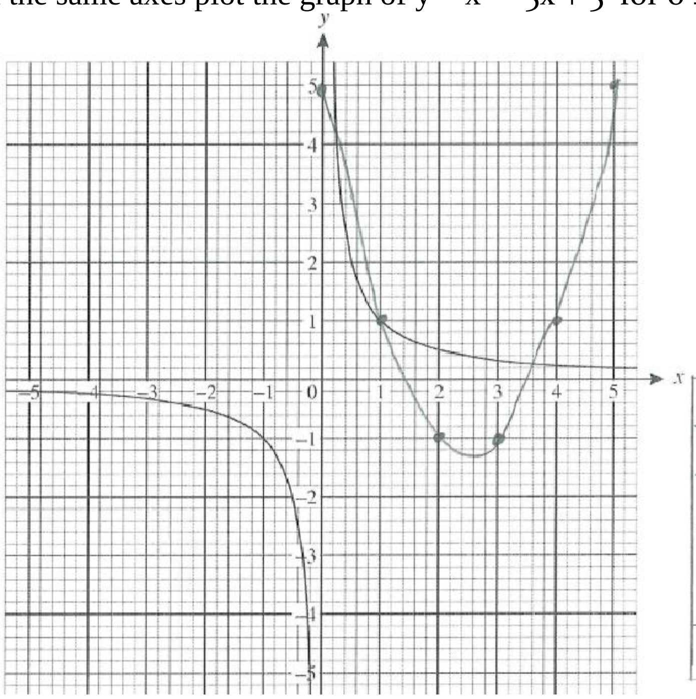
-3 3

(3)

Question 3

The graph below shows the graph of $y = \frac{1}{x}$

On the same axes plot the graph of $y = x^2 - 5x + 5$ for $0 \le x \le 5$



x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$-5x$	0	-5	-10	-15	-20	-25
$+5$	+5	+5	+5	+5	+5	+5
y	5	1	-1	-1	1	5

(4)

Total / 10

8 Transformation of functions

Question 1

The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 & 0 \end{pmatrix}$

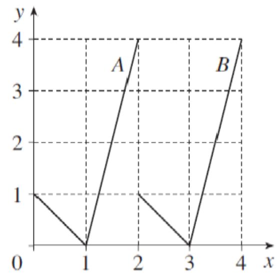
Write down an equation for the translated curve. You need not simplify your answer.

$y = (x-2)^2 - 4$

(2)

Question 2

This diagram shows graphs A and B.



(i) State the transformation which maps graph A onto graph B

A movement of 2 to the right is a translation of $\begin{pmatrix} +2 \\ 0 \end{pmatrix}$

(2)

(ii) The equation of graph A is y = f(x).
Which one of the following is the equation of graph B ?

$y = f(x) + 2$	$y = f(x) - 2$	$y = f(x+2)$	$y = f(x-2)$
$y = 2f(x)$	$y = f(x+3)$	$y = f(x-3)$	$y = 3f(x)$

$f(x-2)$

$f(x)+2$

$\uparrow$ up 2

$f(x+2) \xleftarrow{2} f(x) \xrightarrow{2} f(x-2)$

$\downarrow$ down 2

$f(x)-2$

Answer f(x-2)

(2)

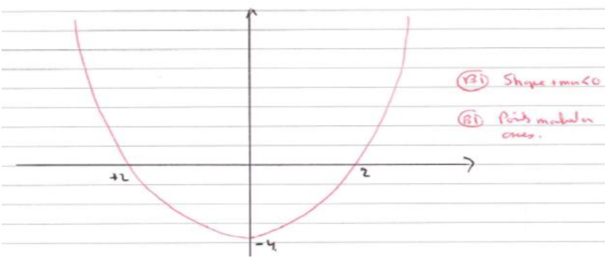
Question 3

(i) Describe the transformation which maps the curve $y = x^2$ onto the curve $y = (x+4)^2$

- Translation B1
- $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ B1 (or 4 units to the left)

(2)

(ii) Sketch the graph of $y = x^2 - 4$

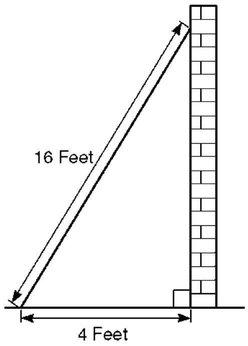


(2)

Total / 10

9 Trigonometric ratios

Question 1
Sidney places the foot of his ladder on horizontal ground and the top against a vertical wall.
The ladder is 16 feet long.



The foot of the ladder is 4 feet from the base of the wall.

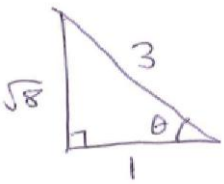
(i) Work out how high up the wall the ladder reaches. Give your answer to 3 significant figures.

$\sqrt{16^2 - 4^2}$
$\sqrt{256 - 16}$ correct substitution (M1)
$\sqrt{240}$
15.49
15.5 (3sf) (A1) (2)

(ii) Work out the angle the base of the ladder makes with the ground. Give your answer to 3 sig fig

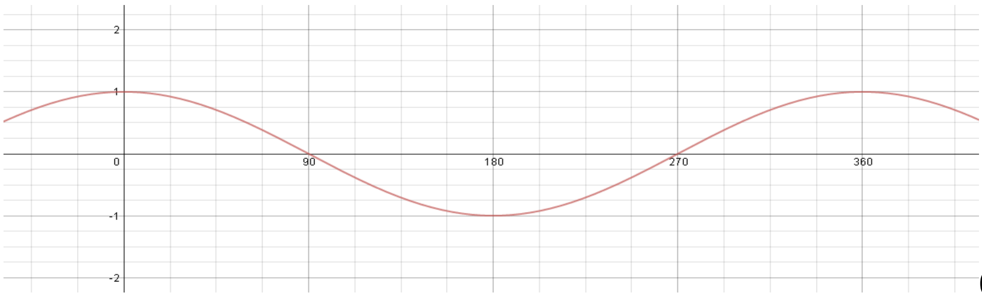
$\cos x = \frac{4}{16}$ correct ratio and substitution (M1)
$\cos x = 0.25$
$x = 75.522$
$x = 75.5°$ (A1) (2)

Question 2
Given that $\cos \Theta = \frac{1}{3}$ and Θ is acute, find the exact value of $\tan \Theta$



$\tan \theta = \frac{\text{opp.}}{\text{Adj}} = \frac{\sqrt{8}}{1} = \sqrt{8}$ (3)

Question 3
Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360°$

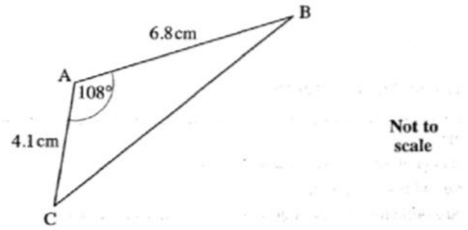


(3)

Total / 10

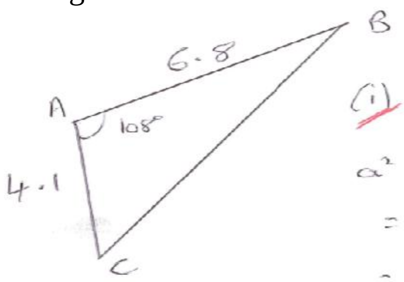
10 **Sine / Cosine Rule**

Question 1



For triangle ABC, calculate

(i) the length of BC



(i) By the Cosine Rule.

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 6.8^2 + 4.1^2 - 2 \times 6.8 \times 4.1 \times \cos 108$

$= 63.05 - - 17.23$

$= 80.28$

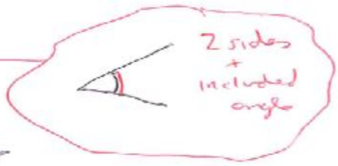
$\Rightarrow a = \sqrt{80.28} = 8.960$

(3)

(ii) the area of triangle ABC

Area of a Triangle

$= \frac{1}{2} ab \sin C$



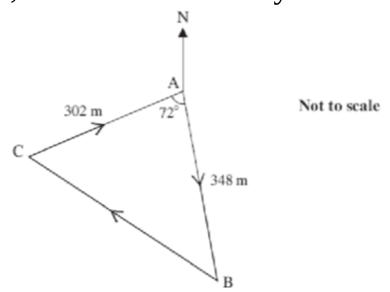
$= \frac{1}{2} \times 4.1 \times 6.8 \times \sin 108$

$= 13.26$

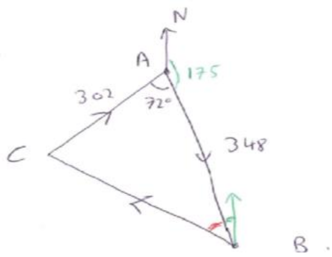
(3)

Question 2

The course for a yacht race is a triangle as shown in the diagram below. The yachts start at A, then travel to B, then to C and finally back to A.



Calculate the total length of the course for this race.



Use the Cosine Rule to find CB

$CB^2 = 302^2 + 348^2 - 2 \times 302 \times 348 \times \cos 72$

$CB = 384$

Total length = 384 + 650 = 1034m

(4)

Total / 10

PiXL
Partners in excellence