The Bridge to A level

Test Yourself Worked Solutions

1 Solving quadratic equations

Question 1

Find the real roots of the equation $x^4 - 5x^2 - 36 = 0$ by considering it as a quadratic equation in x^2

Treat as a quadratic in x^2.

Factorise $(x^2 - 9)(x^2 + 4) = 0$

$\rightarrow$ $(x^2 - 9) = 0$ or $(x^2 + 4) = 0$

$\rightarrow$ $x^2 = 9$ or $x^2 = -4$

$\rightarrow$ $x = \pm 3$ or No real roots

$\rightarrow$ $x = \pm 3$

(4)

Question 2

(i) Write $4x^2 - 24x + 27$ in the form of $a(x - b)^2 + c$

$$4x^2 - 24x + 27$$
$$= 4(x^2 - 6x) + 27$$
$$= 4\left[(x-3)^2 - 9\right] + 27$$
$$= 4(x-3)^2 - 36 + 27$$
$$= 4(x-3)^2 - 9$$

Don't take the factor of 4 out of the constant.

(4)

(ii) State the coordinates of the minimum point on the curve $y = 4x^2 - 24x + 27$.

Minimum point at (3,-9)

(2)

Total / 10

2 Changing the Subject

Question 1

Make t the subject of the formula $s = \frac{1}{2}at^2$

$$s = \frac{1}{2}at^2$$

$$2s = at^2$$

$$\frac{2s}{a} = t^2$$

$$t = \pm\sqrt{\frac{2s}{a}}$$

(3)

Question 2

Make x the subject of $3x - 5y = y - mx$

2)

$$3x - 5y = y - mx$$

$$3x + mx - 5y = y$$

Get terms with x = terms without x.

$$3x + mx = y + 5y$$

$$x(3 + m) = 6y$$

$$x = \frac{6y}{(3+m)}$$

Factorise, then divide by factor

(3)

Question 3

Make x the subject of the equation $y = \frac{x+3}{x-2}$

$$y = \frac{x+3}{x-2}$$

$$y(x-2) = x+3$$

$$xy - 2y = x+3$$

$$xy - x = 2y+3$$

$$x(y-1) = 2y+3$$

$$x = \frac{2y+3}{y-1}$$

(4)

Total / 10

3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines $x + 2y = 5$ and $y = 5x - 1$

$x + 2(5x-1) = 5$
$x + 10x - 2 = 5$
$11x = 7$
$x = \frac{7}{11}$ $y = \frac{35}{11} - 1$ $y = \frac{24}{11}$

(3)

Question 2

The lines $y = 5x - a$ and $y = 2x + 18$ meet at the point $(7,b)$. Find the values of a and b.

$5x - a = 2x + 18$
$35 - a = 14 + 18$
$a = 3$ $b = 35 - 3 = 32$

(3)

Question 3

A line and a curve has the following equations :

$$3x + 2y = 7 \qquad y = x^2 - 2x + 3$$

Find the coordinates of the points of intersection of the line and the curve by solving these simultaneous equations algebraically

Substitute y from 2nd equation into 1st.

$3x + 2(x^2 - 2x + 3) = 7$

$3x + 2x^2 - 4x + 6 = 7$

$2x^2 - x + 6 = 7$

$2x^2 - x - 1 = 0$

Factorise : $(2x+1)(x-1) = 0$

means $2x+1=0$ or $x-1=0$
$2x = -1$ or $x = 1$
$x = -\frac{1}{2}$

when $x = -\frac{1}{2}$:
$(3 \times -\frac{1}{2}) + 2y = 7$
$-1.5 + 2y = 7$
$2y = 8.5$
$y = 4.25$

when $x = 1$
$(3 \times 1) + 2y = 7$
$3 + 2y = 7$
$2y = 4$
$y = 2.$

First point $(-\frac{1}{2}, 4.25)$
Second point $(1 , 2)$.

(4)

Total / 10

4 Surds

Question 1

(i) Simplify $\sqrt{24} + \sqrt{6}$

(i) $\sqrt{24} + \sqrt{6}$

$= \sqrt{4}\sqrt{6} + \sqrt{6}$

$= 2\sqrt{6} + \sqrt{6}$

$= 3\sqrt{6}$

(2)

(ii) Express $\frac{36}{5-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers.

(ii) $\frac{36}{5-\sqrt{7}} = \frac{36}{(5-\sqrt{7})} \times \frac{(5+\sqrt{7})}{(5+\sqrt{7})}$

$= \frac{36 \times (5+\sqrt{7})}{25 - 5\sqrt{7} + 5\sqrt{7} - (\sqrt{7})^2}$

$= \frac{36(5+\sqrt{7})}{18}$

$= 2(5+\sqrt{7})$

$= 10 + 2\sqrt{7}$

To rationalise the denominator of $(x - \sqrt{y})$ top + multiply by bottom $(x + \sqrt{y})$

(3)

Question 2

(i) Simplify $6\sqrt{2}$ x $5\sqrt{3}$ - $\sqrt{24}$

7.i) $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$

$= 30\sqrt{6} - \sqrt{4}\sqrt{6}$

$= 28\sqrt{6}$

(2)

(ii) Express $(2 - 3\sqrt{5})^2$ in the form $a + b\sqrt{5}$, where a and b are integers.

$(2-3\sqrt{5})(2-3\sqrt{5}) = 4 - 6\sqrt{5} - 6\sqrt{5} + 9\times5$

$= 49 - 12\sqrt{5}$. (3)

(3)

Total / 10

5 Indices

Question 1

Find the value of the following.

(i) $\left(\frac{1}{3}\right)^{-2}$

$\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^{2}$

$= 3^{2}$

$= 9$

(2)

(ii) $16^{\frac{3}{4}}$

$16^{3/4} = \left(16^{\frac{1}{4}}\right)^{3}$

$= (2)^{3}$

$= 8$

(2)

Question 2

(i) Find a, given that $a^3 = 64x^{12}y^3$

$a^3 = 64x^{12}y^3$

$= 4^3(x^4)^3y^3$

$= (4x^4y)^3$

$\Rightarrow a = 4x^4y$

(2)

(ii) $\left(\frac{1}{2}\right)^{-5}$

$\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^{5}$

Negative index inverts the fraction

$= 2^{5}$

$= 32$

(2)

Question 3

Simplify $\frac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$

$\frac{16^{\frac{1}{2}}}{81^{3/4}} = \frac{\sqrt{16}}{\sqrt[4]{81}^{3}} = \frac{4}{3^3} = \frac{4}{27}$

(2)

Total / 10

6 Properties of Lines

Question 1

The points A (-1,6), B (1,0) and C (13,4) are joined by straight lines. Prove that AB and BC are perpendicular.

Grad of AB = $\frac{0-6}{1--1}$ = -3

Grad of BC = $\frac{4-0}{13-1}$ = $\frac{1}{3}$

Product of gradients is = $-3 \times \frac{1}{3}$ = -1. Hence AB and BC are perpendicular.

(2)

Question 2

A and B are points with coordinates (-1,4) and (7,8) respectively. Find the coordinates of the midpoint, M, of AB.

Midpoint is $\left(\frac{7+-1}{2}, \frac{8+4}{2}\right)$ = (3, 6)

(1)

Question 3

A line has gradient -4 and passes through the point (2,-6). Find the coordinates of its points of intersection with the axes.

Equation of line is (y - -6) = -4(x – 2) ie y = -4x + 2

$x = 0 \rightarrow y = 2$ Coordinates (0,14)
$y = 0 \rightarrow x = 0.5$ Coordinates (3.5,0)

(4)

Question 4

Find the equation of the line which is parallel to y = 3x + 1 and which passes through the point with coordinates (4,5).

Gradient 3

(y – 5) = 3(x – 4)

$\rightarrow$ y = 3x - 7

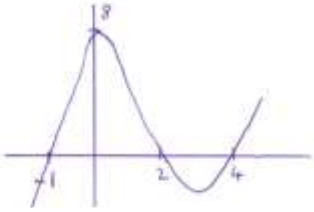
(3)

Total / 10

7 Sketching curves

Question 1

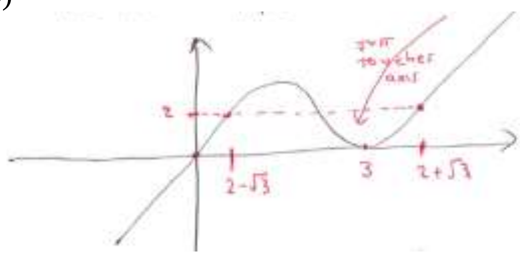
You are given that $f(x) = (x + 1)(x – 2)(x – 4)$. Sketch the graph of $y = f(x)$



(3)

Question 2

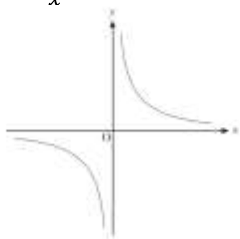
Sketch the graph of $y = x(x - 3)^2$



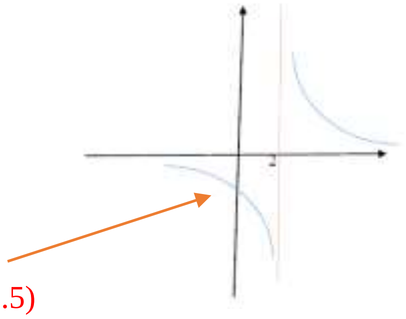
(3)

Question 3

This diagram shows a sketch of the graph of $y = \frac{1}{x}$



Sketch the graph of $y = \frac{1}{x-2}$, showing clearly any points where it crosses the axes.

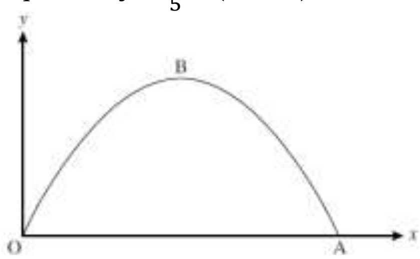


(0,-0.5)

(3)

Question 4

This curve has equation $y = \frac{1}{5}x(10 - x)$. State the value of x at the point A.



$y = \frac{x}{5}(10 - x)$

$x = 10$ B1

(1)

Total / 10

8 Transformation of functions

Question 1

The graph of y = x2 – 8x + 25 is translated by $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$. State an equation for the resultant graph.

$y = x^2 - 8x + 25 - 20$

$\Rightarrow \underline{y = x^2 - 8x + 5}$

(1)

Question 2

$f(x) = x^3 - 5x + 2$

Show that $f(x - 3) = x^3 - 9x^2 + 22x - 10$

$f(x - 3) = (x - 3)^3 - 5(x - 3) + 2$

$= (x^2 - 6x + 9)(x - 3) - 5x + 15 + 2$

$= x^3 - 3x^2 - 6x^2 + 18x + 9x - 27 - 5x + 15 + 2$

$= x^3 - 9x^2 + 22x - 10$

(4)

Question 3

You are given that $f(x) = 2x^3 + 7x^2 - 7x - 12$

Show that $f(x - 4) = 2x^3 - 17x^2 + 33x$

$f(x-4) = (x-4+4)(2(x-4)-3)(x-4+1)$

$= x(2x-8-3)(x-3)$

$= x(2x-11)(x-3)$

$= x(2x^2 - 11x - 6x + 33)$

$= \underline{2x^3 - 17x^2 + 33x}$

(3)

Question 4

You are given that f(x) = (x + 1)(x – 2)(x – 4). The graph of y = f(x) is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

State an equation for the resulting graph. You need not simplify your answer.

(2)

(x + 1 – 3)(x – 2 – 3)(x – 4 – 3)

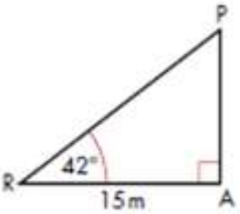
ie (x -2)(x – 5)(x – 7)

Total / 10

9 Trigonometric ratios

Question 1

AP is a telephone pole. The angle of elevation of the top of the pole from the point R on the ground is 42°as seen in the diagram.



Calculate the height of the pole. Give your answer to 3 significant figures.

Tan 42° = $\frac{opp}{adj}$ (M1)

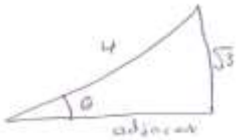
0.9004 = $\frac{height\ of\ pole}{15}$ (M1)

13.5(06) m = height of pole (A1)

(3)

Question 2

Given that sin Θ = $\frac{\sqrt{3}}{4}$, find in surd form the possible values of cos Θ.



sin θ = $\frac{\sqrt{3}}{4}$

By Pythagoras' $4^2 = (\sqrt{3})^2 + adjacent^2$

adjacent = $\sqrt{13}$

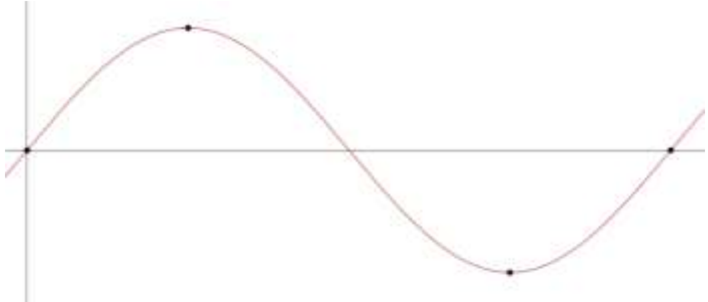
⇒ cos θ = $\pm \frac{\sqrt{13}}{4}$

N.B. 2 values because

(3)

Question 3

The graph of y = sin x for $0 \le x \le 360°$ is shown below.



What are the coordinates of the 4 points labelled on the graph?

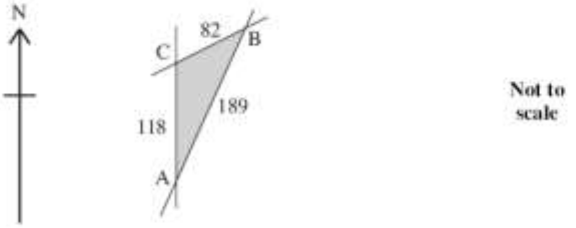
(……0…, …0…)
(90…, ……1…)
(270…, …-1……)
(360…, ……0…)

(4)

Total / 10

10 Sine / Cosine Rule

Question 1
This diagram shows a village green which is bordered by 3 straight roads AB, BC and CA. The road AC runs due North and the measurements are shown in metres.



(i) Calculate the bearing of B from C, giving your answer to the nearest 0.1°

$$\cos C = \frac{82^2 + 118^2 - 189^2}{2 \times 82 \times 118} = -0.778$$

$$C = \cos^{-1}(-0.778)$$
$$= 141.1°$$

$$\text{Bearing} = 180 - 141.1° = 38.9°$$

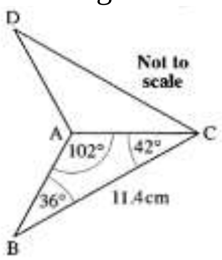
(4)

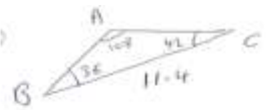
(ii) Calculate the area of the village green.

$$\text{Area} = \tfrac{1}{2}\, bc \sin C$$
$$= \tfrac{1}{2} \times 82 \times 118 \times \sin 141.1°$$
$$= 3032$$

(2)

Question 2
This diagram shows a logo ABCD. It is symmetrical about AC.
Find the length of AB and hence find the area of the logo





By the Sine Rule

$$\frac{AB}{\sin 42} = \frac{11.4}{\sin 108}$$

$$\Rightarrow AB = \frac{11.4 \times \sin 42}{\sin 108} = 7.798$$

$$\text{Area of logo} = 2 \times \text{Area of Triangle}$$
$$= 2 \times \tfrac{1}{2}\, ac \sin B$$
$$= 11.4 \times 7.798 \times \sin 36$$
$$= 52.2$$

(4)

Total / 10